

**ADITYA UNIVERSITY**

B.Tech – I Semester End Examinations Regular – Dec 2024

**LINEAR ALGEBRA AND CALCULUS**

(Common to CE, EEE, ME, ECE, CSE, IT, Min.E, PT, CSE (DS) &amp; AIML)

Time: 3 hours

Max. Marks: 50

Answer ONE question from each unit

All Questions Carry Equal Marks

All parts of the questions must be answered at one place only

**UNIT-I**

- 1 a Find the rank of the matrix  $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$  by reducing it to the echelon form. L2 CO1 [5M]

- b Solve  $2x - y + 3z = 9$ ,  $x + y + z = 6$ ,  $x - y + z = 2$ , using Gauss elimination method. L3 CO1 [5M]

(OR)

- 2 a Find the inverse of the matrix  $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ , using Gauss-Jordan method. L3 CO1 [5M]

- b Solve the following homogeneous system of equations  $x + y - 3z + 2w = 0$ ,  $2x - y + 2z - 3w = 0$ ,  $3x - 2y + z - w = 0$ ,  $-4x + y - 3z + w = 0$ . L3 CO1 [5M]

**UNIT-II**

- 3 Verify Cayley-Hamilton theorem and find the inverse of the matrix  $\begin{bmatrix} 1 & 1 & 2 \\ 9 & 2 & 0 \\ 5 & 0 & 3 \end{bmatrix}$ . L3 CO2 [10M]

(OR)

- 4 Reduce the quadratic form  $x^2 + y^2 - z^2 + 6xy + 4yz - 2zx$  to the normal form by orthogonal transformation. L3 CO2 [10M]

**UNIT-III**

- 5 a For  $f(x) = e^x$  and  $g(x) = e^{-x}$ . Determine the point c in the interval  $[1, 3]$ , using Cauchy's Mean Value Theorem. L3 CO3 [5M]

- b If  $f(x, y) = \sin(xy) + x^2 \log_e y$ , then find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$ . L2 CO3 [5M]

(OR)

- 6 a Using Maclaurin's series, show that  $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$  L3 CO3 [5M]

- b Evaluate  $\lim_{\substack{x \rightarrow 2 \\ y \rightarrow 3}} \frac{x^8 + y^8}{2x^2y}$ . L2 CO3 [5M]

**UNIT-IV**

- 7 a Express the Taylor's series expansion for  $f(x, y) = x^4y + 3y - 2$  in powers of  $(x - 1)$  &  $(y + 2)$ . L3 CO4 [5M]

- b A rectangular box open at the top is to have a volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. L3 CO4 [5M]

(OR)

- 8 a If  $u = xy + yz + zx$ ,  $v = x^2 + y^2 + z^2$  and  $w = x + y + z$ . Determine whether there is a functional relationship between  $u$ ,  $v$ ,  $w$  and if so, find it. L3 CO4 [5M]
- b Determine the maxima and minima of  $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ . L2 CO4 [5M]

## UNIT-V

- 9 By changing the order of integration, evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy dx$  L3 CO5 [10M]

(OR)

- 10 a Find the area enclosed by the parabolas  $y = x^2$  and  $x = y^2$ . L2 CO5 [5M]
- b Evaluate the following integral by transforming into polar co-ordinates  $\int_0^a \int_0^{\sqrt{a^2-x^2}} y \sqrt{x^2 + y^2} dy dx$ . L3 CO5 [5M]

\*\*\*\*\*