



ADITYA UNIVERSITY

B.Tech – I Semester End Examinations Regular (B.Tech2024)

LINEAR ALGEBRA AND CALCULUS

(Common to CE, EEE, ME, ECE, CSE, IT, Min.E, PT, DS & AIML)

Time: 3 hours

Max. Marks: 50

Answer ONE question from each unit

All Questions Carry Equal Marks

All parts of the questions must be answered at one place only

UNIT-I

- 1 a Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$, by reducing it to the echelon form. L2 CO1 [5M]
- b Solve $10x_1 + x_2 + x_3 = 12$, $x_1 + 10x_2 - x_3 = 10$, $x_1 - 2x_2 + 10x_3 = 9$ using Gauss elimination method. L3 CO1 [5M]

(OR)

- 2 a Find the inverse of the matrix $\begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ using Gauss-Jordan method. L2 CO1 [5M]
- b Solve the following Homogeneous system of equations L3 CO1 [5M]
 $x + y - 3z + 2w = 0$, $2x - y + 2z - 3w = 0$,
 $3x - 2y + z - w = 0$, $-4x + y - 3z + w = 0$.

UNIT-II

- 3 a Find the Eigen values and the corresponding Eigen vectors of the matrix L3 CO2 [5M]
 $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$.
- b Verify Cayley-Hamilton theorem and find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. L3 CO2 [5M]

(OR)

- 4 Reduce the quadratic form $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$ to the normal form by orthogonal transformation. L3 CO2 [10M]

UNIT-III

- 5 a Verify Cauchy's mean value theorem for $f(x) = \sin x$ and $g(x) = \cos x$ on $[0, \frac{\pi}{2}]$. L3 CO3 [5M]
- b Find first and second order partial derivatives of $f = x^3 + y^3 - 3axy$ and verify $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ L2 CO3 [5M]

(OR)

- 6 a Using Maclaurin's series, show that L3 CO3 [5M]
$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

b If $u = \tan^{-1}\left[\frac{2xy}{x^2-y^2}\right]$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ L2 CO3 [5M]

UNIT-IV

- 7 a Expand e^{xy} in the neighbourhood of (1,1). L3 CO4 [5M]
b Find the maximum and minimum values of L3 CO4 [5M]
 $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4.$

(OR)

- 8 a Show that the functions $u = \frac{x}{y}$, $v = \frac{x+y}{x-y}$ are functionally related. L3 CO4 [5M]
Find the relation between them.
b Find the maximum and minimum values of L3 CO4 [5M]
 $x + y + z$ subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1.$

UNIT-V

- 9 a Evaluate $\iint xy(x+y)dx dy$ over the region R bounded by $y = x^2$ and $y = x$. L3 CO5 [5M]
b Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar co-ordinates. L3 CO5 [5M]

(OR)

- 10 a Evaluate $\iint r \sin \theta dr d\theta$ over the cardioid $r = a(1 - \cos\theta)$ above the initial line. L3 CO5 [5M]
b Evaluate the integral by changing the order of integration L3 CO5 [5M]
 $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx.$